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ABSTRACT

This brochure explains a number of aspects of the standards established by the National Council of Teachers of Mathematics for mathematics education. The three sets of standards are: (1) Curriculum and Evaluation Standards; (2) Professional Standards for Teaching Mathematics; and (3) Assessment Standards for School Mathematics. Contained in the brochure is an explanation of the rationale for having standards for mathematics education; a discussion of the nature of standards and their application in other professions; an overview of each set of standards; a description of the components of the vision for change in mathematics education; details about thought processes and decision making skills that students need and how those skills change as we advance further into the information age; and a discussion of how schools and programs move towards active student learning, environments that support learning, and ongoing assessment based on multiple sources. (DDR)

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MATHEMATICS

AN INTRODUCTION TO THE NCTM STANDARDS

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MATHEMATICS

AN INTRODUCTION TO THE NCTM STANDARDS

Charleen Herrick, Bell Park Portrait Studio

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

STANDARDS FOR TODAY'S MATH

More than at any other time in history, society is placing demands on citizens to interpret and use mathematics to make sense of information and complex situations. Computers and other technologies have increased our capacities for dealing with numbers, for collecting, organizing, representing, and analyzing data, and for analyzing mathematical models symbolically and graphically. Tables, lists of numbers, graphs of data, and statistics summarizing information occur in every form of the media.

To be well informed as adults and to have access to desirable jobs, students today require an education in mathematics that goes far beyond what was needed by students in the past. All students must develop and sharpen their skills, deepen their understanding of mathematical concepts and processes, and hone their problem-solving, reasoning, and communication abilities while using mathematics to make sense of, and to solve, compelling problems. This increased emphasis on the broad spectrum of mathematics and its applications is addressed in each of three *Standards* documents prepared by the National Council of Teachers of Mathematics (NCTM). The common thread in these documents is that all students need a deep understanding of mathematics; for this to occur, rigorous mathematical content must be organized, taught, and assessed in a problem-solving environment.

Hands on activities taught in and outside the classroom, such as blowing a spherical bubble through a noncircular hole, can help promote meaningful learning.

EMATICS

Envision a classroom where students are actively engaged in learning important mathematics. The students use mathematics to investigate, conjecture, reason logically, solve problems, and communicate their ideas. In this kind of classroom setting, all students have the opportunity to develop what the NCTM calls "mathematical power." To help states, provinces, local school districts, individual teachers, and parents put such a mathematics education in place, the NCTM's membership has formulated and articulated in three documents a set of standards for curriculum, instruction, and assessment.

As students work and learn together, they come to value diversity, recognize individual strengths and skills, and appreciate cooperation and interaction.

WHAT ARE STANDARDS?

From pharmaceuticals to building construction, from auto manufacturing to food processing, most institutions set standards for three reasons: to indicate goals, to examine current practice, and to ensure quality. Education is no different. NCTM's *Curriculum and Evaluation Standards for School Mathematics*, published in 1989, *Professional Standards for Teaching Mathematics*, published in 1991, and *Assessment Standards for School Mathematics*, published in 1995, provide goals and principles against which curriculum, instruction, and assessment practices and frameworks can be examined. Yet, the *Standards* are not how-to-do-it books. Instead, they are meant to stimulate policymakers, parents, teachers, the community, school boards, and administrators to examine and improve mathematics programs at all levels in ways consonant with local beliefs.

Each *Standards* document was written by a team of NCTM members whose collective knowledge and professional responsibilities reflect the diversity of the membership of the Council. These teams included classroom teachers, supervisors, educational researchers, teacher educators, and mathematicians. Thousands of others, both NCTM members and nonmembers including parents and business leaders, reviewed and critiqued each document in draft form, so that the final versions represent a broad consensus.

A QUICK LOOK AT EACH SET OF

NCTM's *Curriculum and Evaluation Standards for School Mathematics* describes the mathematics content that all students should know and be able to do. The document is divided into grade levels K-4, 5-8, and 9-12, with each level having twelve to fourteen standards.

These standards identify the basic skills and understandings students should have in number and number theory, geometry, measurement, probability and statistics, patterns and functions, discrete mathematics, algebra, and beyond. While students are mastering such important basic skills, they must also—

- learn to value mathematics;
- become confident in their ability to do mathematics;
- become mathematical problem solvers;
- learn to communicate mathematically;
- learn to reason mathematically.

Given these content goals, the *Professional Standards for Teaching Mathematics* illustrates ways that teachers can structure classroom activities to encourage such learning. In four major areas, decisions by teachers strongly influence students' opportunities to make sense of mathematics:

- Choosing worthwhile mathematical tasks
- Establishing and promoting classroom discussion
- Creating an environment for learning
- Analyzing one's own teaching, including the efficacy of assessing students' learning

The most recent set of standards, *Assessment Standards for School Mathematics*, provides a set of principles for teachers and others to use in examining assessment practices. Educators must ensure that assessment reflects the mathematics that all students need to know and be able to do. Those responsible for mathematics education must be able to draw valid inferences about students' mathematics learning, so that assessment results can be used to modify the teaching process as needed. Moreover, assessment must promote equity, so that all students have the opportunity to demonstrate their mathematical understanding and all teachers work to help students when understanding is not yet complete. For all this to occur, assessment must be an open, coherent process.

Notice that mathematics is the centerpiece of each of the *Standards* documents. What students should know and be able to do and what kinds of mathematical tasks and experiences will help students reach these goals are the heart of the messages articulated in the three documents.

National Education Association, Joe Dido

STANDARDS

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Many real life activities require only simple tools. The daily newspaper, for example, can be a rich source of data for a project.

National Education Association, Joe Dello

Concepts of capacity and volume are reinforced in a hands-on activity.

UNDERSTANDING THE VISION

The vision for school mathematics represented by the *Standards* documents implies four major changes or shifts in educational practice:

- ***In curriculum***, a shift toward a deeper study of important mathematics and its uses
- ***In learning***, a shift toward active student engagement with a variety of mathematical problem situations and a variety of mathematical tools for solving these problems
- ***In teaching***, a shift toward creating stimulating learning environments in which all students are challenged to reach their full potential
- ***In assessment practices***, a shift toward assessment that is ongoing and based on many sources of evidence

TOWARD A DEEPER STUDY OF IMPORTANT MATHEMATICS

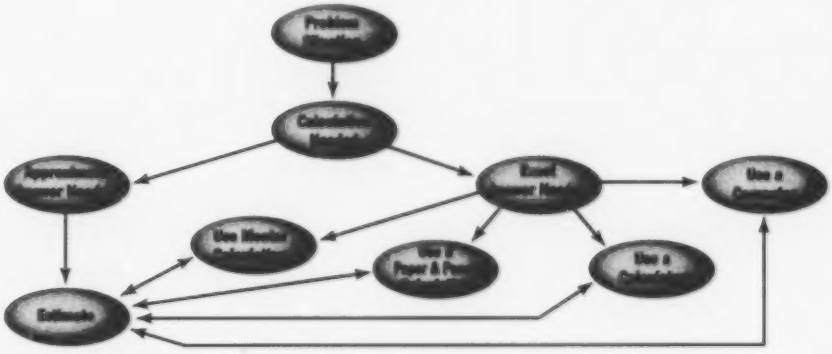
The *Standards* documents reflect a shift in the value that the world outside schools places on thinking and problem solving. The pervasive use of quantitative reasoning in all aspects of everyday life and work makes mathematics central to a new way of thinking about what a literate adult must know. The use of mathematics as a tool, not just for calculating but also for inventing and analyzing in the workplace, has increased dramatically in importance as our society has moved through the industrial age into the information age.

Procedural skills are still important. Equally important, however, is that students learn to analyze a situation and choose a procedure or a set of procedures that are likely to furnish insight into the problem situation. The student thus becomes the director, the engineer, the inventor of a way to solve a problem as well as the problem solver.

For students to develop this deeper level of understanding, their knowledge must not be fragmented and isolated but rather connected to a variety of ideas and skills. Identifying common ways of thinking, common procedures, and common concepts across topic areas and grade levels in mathematics is a major goal of the *Curriculum Standards*. In addition to making connections within mathematics, these *Standards* maintain that mathematics should be connected to other subjects taught in school, such as social studies, art, music, and English and language arts, as well as to situations outside the classroom.

Most adults think of school mathematics as arithmetic followed by algebra and geometry in the high school years. They remember practicing the addition, subtraction, multiplication, and division of numbers in various forms. The *Curriculum and Evaluation Standards* promotes computational skills, but from a larger perspective. Sometimes paper-and-pencil computation is best. Sometimes mental mathematics or an estimate will suffice. Other times, a computer or calculator is the best tool.

The diagram below shows the thought process and decision-making skills students need when they approach problem situations.



This kind of instruction and learning is clearly different from instruction that focuses on memorizing facts alone. The *Standards* documents call for computational knowledge that is gained through problem contexts and strengthened through practice and application to problem situations. Strong evidence from research on students' mathematics learning indicates that conceptual approaches to computation instruction result in high achievement and retention and a reduction in the amount of time children need to master computational skills. Furthermore, many of the errors children typically make are less prevalent. This evidence implies that classroom instruction in which students are expected to think about their answer and discuss whether the results from a computation make sense and why will be more powerful than instruction that focuses on memorizing facts alone.

In classrooms having this expectation about computational proficiency, students consistently ask such questions as these:

- *Can I add, subtract, multiply, or divide these numbers in my head?*
- *Can I make a reasonable estimate of the results from this computation?*
- *Do I need an exact answer?*
- *Do I need a calculator?*

Calculators and computers can enhance mathematical understanding and help improve mental computation and estimation. They are important tools for developing concepts of algebra throughout the grades. Patterns and relationships come to life with spreadsheets using a computer. Students can operate with numbers, find their own patterns, and conjecture how a change in one number influences change in another. Understanding the relationship among ordered pairs of numbers is critical for making sense of algebra. The power to try multiple examples, record, observe, and make predictions from the results can be dramatically enhanced by capitalizing on the speed and accuracy of calculators and computers. All these uses of technology have significant implications for instructional practices.

The teacher is the key to changing the ways mathematics is taught and learned in schools. The teacher brings life to the classroom by facilitating investigations of a variety of important topics and problem situations and ensuring that students understand the mathematics involved. The teacher of mathematics should pose tasks that are based on—

- sound and significant mathematics;
- their knowledge of students' understandings, interests, and experiences;
- their knowledge of the range of ways that diverse students learn;
- engaging their students' intellect;
- developing their students' mathematical understandings and skills.

The teacher decides what should be taught and how it should be taught. This decision-making process is ongoing. In addition to preparation before class, teachers take cues from their students' responses to modify, extend, or deepen a lesson. Teachers use assessment results to make decisions when planning lessons.

Assessment should reflect the mathematics that all students need to know and be able to do. If students are to take the right messages from classroom practice about what is expected of them and what is valued, assessments must mirror an emphasis on challenging problem

situations. Assessment should present all students with opportunities to formulate problems, reason mathematically, make connections among mathematical ideas, and communicate about mathematics.

What should be clear in the *Standards* documents is that the curriculum, teaching, and assessment of students' progress cannot be considered separately. Each document contributes to the message that all students should have opportunities to investigate and master important mathematics and its uses.

Technology opens up avenues of learning that are not easily accomplished with pencil-and-paper.

TOWA

Today's classroom must mirror the working world if students are to become productive members of society.

RD ACTIVE STUDENT LEARNING

A fundamental assumption in *Standards*-based education is that students should be actively involved in the learning process. What students learn in mathematics depends to a great degree on the context and ways in which they learn it.

This is apparent in the language that is used throughout the *Standards*. For example, the *Curriculum and Evaluation Standards* states that the study of mathematics should emphasize problem solving so that students can—

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from everyday mathematical situations;
- develop and apply strategies to solve a wide variety of problems;
- verify and interpret results with respect to the original problem;
- acquire confidence in using mathematics meaningfully.

The verbs used here—*investigate, formulate, apply*—suggest that students are actively engaged in solving problems, not passively watching the teacher solve problems on a chalkboard.

This vision of active learning has implications for teaching that are described in the *Professional Teaching Standards*. This document discusses the teacher's and the students' roles in classroom discourse. Through such discourse, students actively engage in making conjectures, seek evidence to support, refine, or reject those conjectures, and create and communicate their evidence and arguments to support their findings. In a *Standards*-based classroom, we see

- teachers asking questions to guide discussion and to model ways of constructively critiquing others' ideas;
- students working individually, in small groups, and in whole-class discussions, using tools such as cubes, grid paper, and computers to make sense of interesting and important mathematical situations and problems;
- students engaged in powerful mathematical thinking as they come to appreciate their own growing knowledge.

TOWARD CREATING ENVIRONMENTS THAT SUPPORT LEARNING

Active student learning depends on active classroom instruction. Teachers consistently ask probing questions and listen as students describe their problem-solving strategies. By observing and listening, teachers can judge the accuracy and depth of students' learning. Teachers do convey knowledge and develop their students' skills, but the *Standards* advocate a major shift in the teacher's habits and actions in the classroom. The teacher's role is not merely to present concepts and illustrate procedures but to ensure that students learn. Active student learning is possible only in classrooms where interaction and discussions are routine.

Some characteristics of an exemplary classroom are high expectations for all students, varied opportunities for learning, and active involvement with topics in genuine problem situations. At the heart of this shift in teaching is the Equity Standard discussed in the *Assessment Standards*. Adherence to an equity standard means that all students, including those with special needs or talents, are expected to reach high levels of accomplishment. It also means that each student is given opportunities to reach those levels and the necessary support to do so.

Dynamic discussion in the classroom will take advantage of the diversity of background, experiences, and previous knowledge that students bring to mathematics classes. Teachers must establish the classroom as a learning community where students gain insight, build experience, and expand their knowledge.

One aspect of the teacher's role is to provoke students' reasoning about mathematics through the tasks they assign and the questions they ask. For example, teachers should regularly follow students' statements with "Why?" or by asking them to explain their thinking. Doing this regularly, whether or not the students' statements are correct, is an important part of establishing discourse centered on mathematical reasoning.

Teachers should expect and encourage students to participate, and they should establish an atmosphere of mutual respect. Such an environment does not mean that

Paper-folding activities reinforce basic principles of geometry for students.

students and teachers accept all answers and ideas as worthwhile and valid. Rather, the intent of this learning environment is to develop students' understanding of the mathematics, not merely to make them feel good about participating.

Serious mathematical thinking takes time as well as intellectual courage and skills. A learning environment that supports problem solving must allow time for students to puzzle, to be confused, to try alternative approaches, and to confer with one another and with the teacher. Furthermore, tasks that require reasoning and problem solving cannot be judged by the speed, pace, and quantity of students' work.

Each *Curriculum Standard* is not intended to stand alone as a chapter in a text or a particular unit of instruction; rather, learning activities should incorporate topics and ideas from across the standards. And, where appropriate, mathematics should be studied in contexts that give the ideas and concepts meaning.

TOWARD ASSESSMENT THAT IS BASED ON MULTIPLE SOURCES

What do students know about mathematics, and what kind of mathematics can they do? This is an important question for parents, teachers, potential employers, college admissions officers, and the general public. Students themselves need to know how they are progressing toward valued learning goals. Assessment is the process by which such questions can be answered. Assessment consists of gathering evidence so that teachers, students, and other interested parties can make inferences from the evidence about what students know and can do. Evidence may come from observing students, asking them questions and listening to them, or examining what they produce. Assessment can be an opportunity for both teaching and learning to occur instead of being an interruption of the teaching and learning process.

The *Professional Teaching Standards* also describes how the lines between teaching and assessment can blur. In the following example, a teacher uses an ordinary written test as an opportunity to interview students about what they know. The vignette illustrates that the teacher does not stop teaching the concepts when the test is over but uses the students' responses to modify his teaching and design further learning activities.

A class of second graders has just finished working on addition and subtraction with regrouping. On a written test, many of them "forget" to regroup when needed in subtraction. Instead, they do this:

$$\begin{array}{r} 50 \\ -\ 38 \\ \hline 28 \end{array}$$

The teacher decides to sit down with the children individually and let them talk through their thinking as they illustrate answers with Popsicle sticks from their kits of materials.

When they use the Popsicle sticks, they find that their answers don't make sense, and they revise them to match what they do with the sticks. The teacher had assumed that if they "saw" the concepts by actually touching the objects, they would understand. He now thinks that maybe he didn't do enough to help them build the links between the concrete model and the algorithm. He starts wondering what he could do to help them make that connection better. He hypothesizes that maybe they know how to regroup but may not understand why or when regrouping is necessary. He decides to make up a worksheet with examples where regrouping is

ONGOING AND

Technology, such as calculators and computers, should be used as tools to support computation and to develop conceptual understanding.

necessary and some where it is not and have the students discuss whether or not they would have to regroup in each case and the reason for their decision.

The *Standards* emphasize the need to make use of multiple sources of information about what students are learning. These sources should encompass tasks that—

- demand different kinds of mathematical thinking;
- present the same mathematical concept or procedure in different contexts, formats, and problem situations.

When teachers find that students perform in consistent ways on different kinds of tasks that demand a range of mathematical thinking, they can feel more confident in the accuracy of their judgments. The *Assessment Standards* provides help and stimulation to begin the process of creating assessment procedures that support the development of mathematical understanding for all students.

ACHIEVING THE VISION

The goal of the *Standards* is to challenge all who are engaged in the mathematics education of students and those who set policy that affects the working conditions and expectations for teachers of mathematics to make a commitment to establish environments in which all students learn to use mathematics effectively and confidently. This commitment carries with it the responsibility to examine all aspects of the system: curriculum, teaching and learning, and assessment. This commitment also includes giving teachers access to the tools and materials they need to engage their students in challenging mathematics as well as supporting continued professional development.

To reach the goals of the *Standards*, schools must become places where students actively engage in investigations of important and useful mathematics in compelling contexts and problems, where classrooms support learning, and where assessment enhances the learning process for both teacher and student. These major shifts in mathematics education cannot be accomplished by teachers alone. They require the engagement, commitment, and efforts of students, parents, school administrators, local community leaders, and policymakers at every level of government.

Realizing this vision is the challenge. The reward is students who will become literate and numerate and who will enter the workforce with intellectual tools and ways of thinking mathematically that will maximize their job options and their effectiveness on the job. The three NCTM *Standards* documents can be used by all who take up this challenge so as to examine current practices in order to build and monitor powerful and effective mathematics programs for all students.

An interactive learning environment can benefit all students

NCTM is pleased to make additional copies of *Mathematics: An Introduction to the NCTM Standards* (stock #628) available to you and your organization or school.

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